EE 230
Lecture 6

- Linear Systems
  - Poles/Zeros
  - Stability
- Circuit Analysis (Review)
- Amplifiers

Quiz 5

A system has the transfer function T(s)
Determine the poles of the system.

$$T(s) = \frac{2 + \frac{1}{s}}{6 + s + \frac{8}{s}}$$

And the number is ?

1

3

8

5

4

2

6

9

7

And the number is ?

1 3 8

5 4

4

2 6

9 7

Quiz 5

A system has the transfer function T(s)
Determine the poles of the system.

$$T(s) = \frac{2 + \frac{1}{s}}{6 + s + \frac{8}{s}}$$

Solution:

$$T(s) = \frac{2 + \frac{1}{s}}{6 + s + \frac{8}{s}} = \frac{2\left(s + \frac{1}{2}\right)}{s^2 + 6s + 8}$$

$$T(s) = \frac{2\left(s + \frac{1}{2}\right)}{s^2 + 6s + 8} = \frac{2\left(s + \frac{1}{2}\right)}{(s+2)(s+4)}$$

Poles at s = -2 and s = -4

Review from Last Time

For a linear network with a finite number of lumped elements, T(s) can always be expressed as a rational fraction in s with real coefficients

$$T(s)=\frac{\sum_{k=1}^{m} a_k s^k}{\sum_{k=1}^{n} b_k s^k}$$

This can be equivalently expressed in factored form as

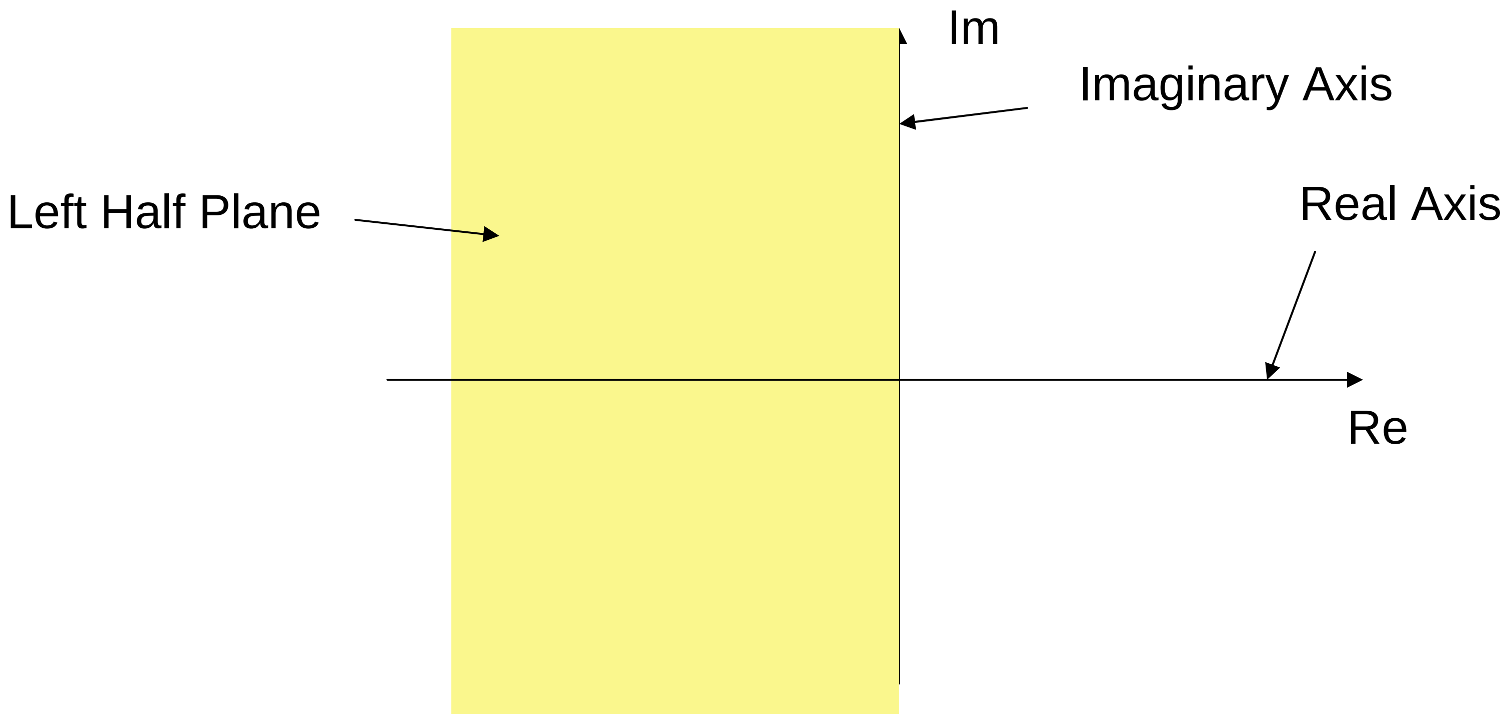
$$T(s)=H\frac{\prod_{k=1}^{m}(s-z_k)}{\prod_{k=1}^{n}(s-p_k)}$$

$\{z_k \quad k=1...m\}$ termed the zeros of $T(s)$

$\{p_k \quad k=1...n\}$ termed the poles of $T(s)$

Review from Last Time

Any network comprised of R's, C's, and L's will have poles in the left half-plane (real part of all poles is negative)



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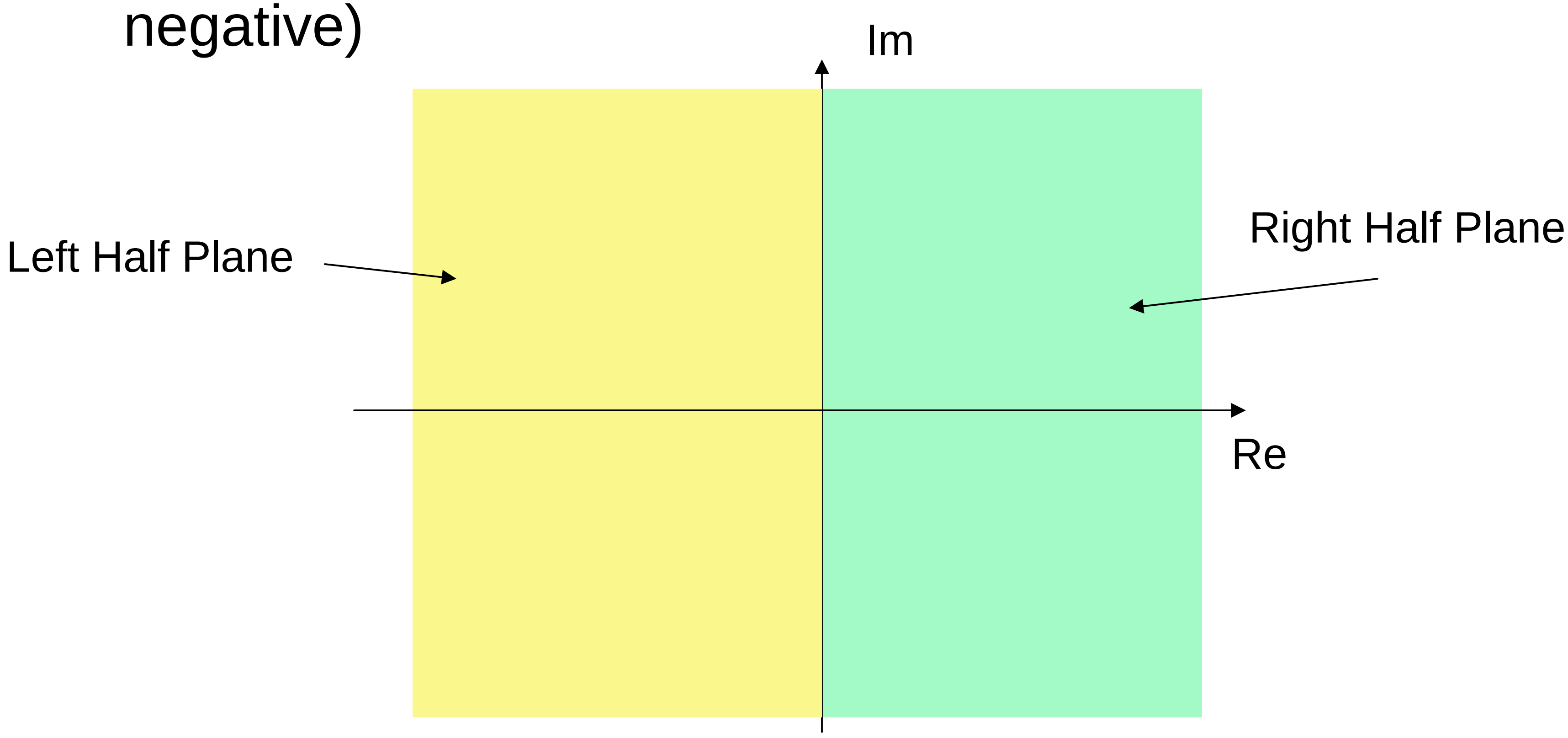
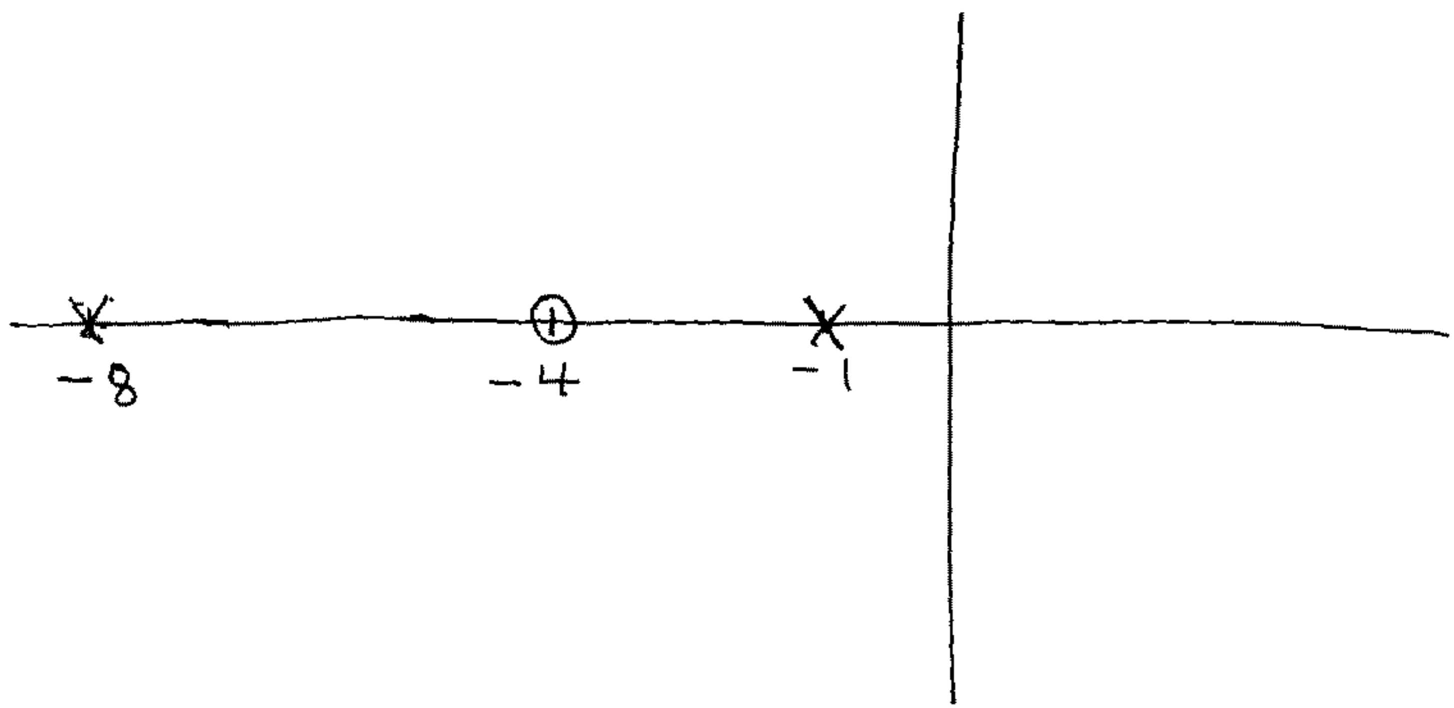


Pole-Zero Plots

- Plots of the poles and zeros in the complex plane (X ⤳ poles, O ⤳ zeros)

Example: $z = -4$, $P_1 = -1$, $P_2 = -8$



Example $T(s) = \dfrac{s-1}{s^2+2s+2}$

Determine poles and zeros and plot in complex plane

$z_1 = 1$

$p = ?$

$s^2+2s+2=0 \Rightarrow s = \dfrac{-2 \pm \sqrt{2^2-(4)(2)}}{2}$

$$s = \frac{-2 \pm \sqrt{-4}}{2}$$

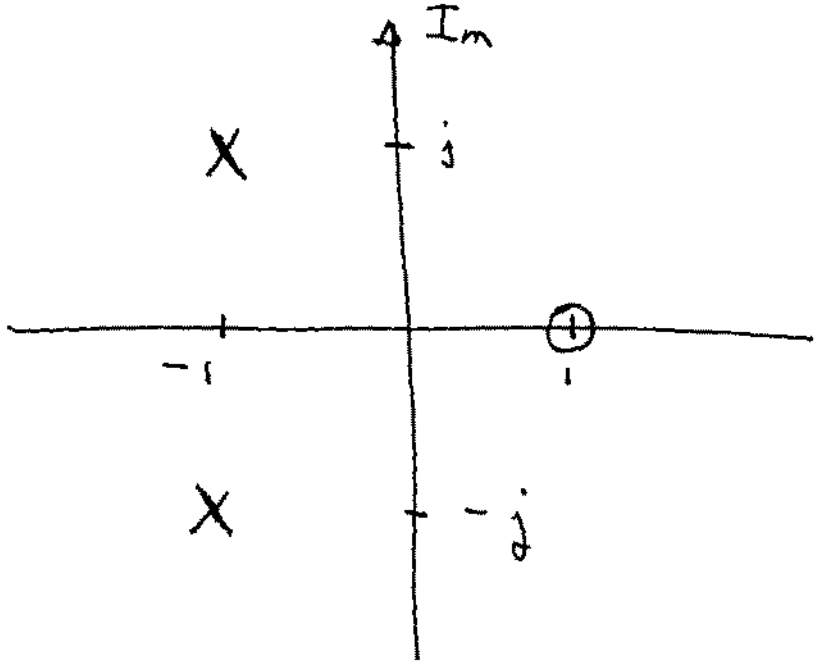
$$s = -1 \pm j$$

$$s = -1 \pm j$$

$$P_1 = -1 + j$$

$$P_2 = -1 - j$$

$$Z_1 = 1$$



Example: $$T(s) = \frac{4s+1}{s^5 + s^4 + 2s^3 + 3s^2 + 2s + 1}$$

Zeros : $\{-\frac{1}{4}\}$

poles : - 5 in number

- closed form expression does not exist for polynomials of order 5 or higher

Theorem: A system is stable iff **all** poles lie in the left half-plane

What is "stable"?

- A linear system is stable iff any bounded input will result in a bounded output

- A system is stable iff the output due to any appropriately small input does not cause the output to go to $\pm\infty$ and does not create an output that persists indefinitely

Examples:

$$T(s) = \frac{1}{s+1}$$

pole at $p = -1$

$\therefore$ system is stable

step response: $r(t) = F + (I - F)e^{-t/\tau} = e^{-t}$

$$T(s) = \frac{1}{s-1}$$

pole at $p = 1$

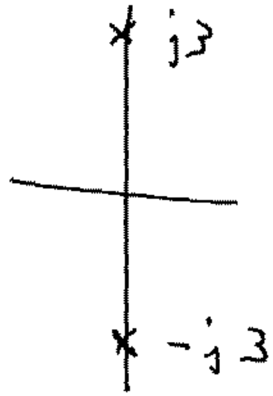
$\therefore$ system is unstable

It can be shown that the step response is

$$r(t) = e^{t}$$

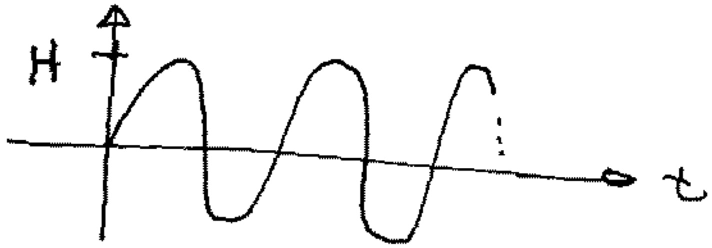
$r(t)$ diverges to ∞ !

Example: $T(s) = \dfrac{4}{s^2+9}$ Poles: $\{-j3, +j3\}$



It can be shown that the step response will include a term

$$r(t) = H \sin 3t$$



$\therefore$ output persists indefinitely

Is stability good or bad?

- depends on what is desired

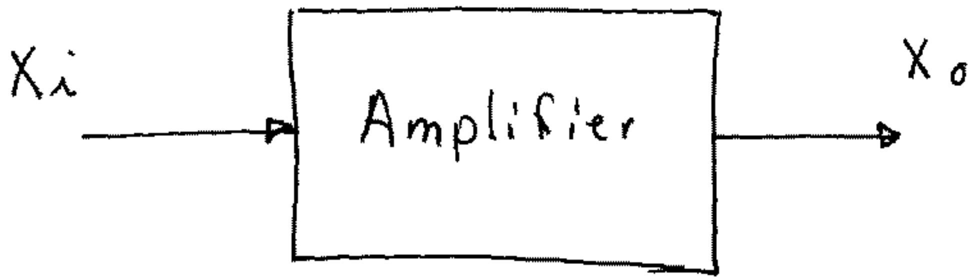
Is instability good or bad?

- depends upon what is desired

End of Lecture 6

Amplifiers

Amplifiers



An ideal amplifier is linear and has a frequency independent transfer function that does not change with source or load impedance.

Ideally: $X_o = K X_i$

K is termed amplifier gain

$K = T(s)$. Often K ">" 1

Types of Amplifiers
(variables of Interest $\{v, i\}$)

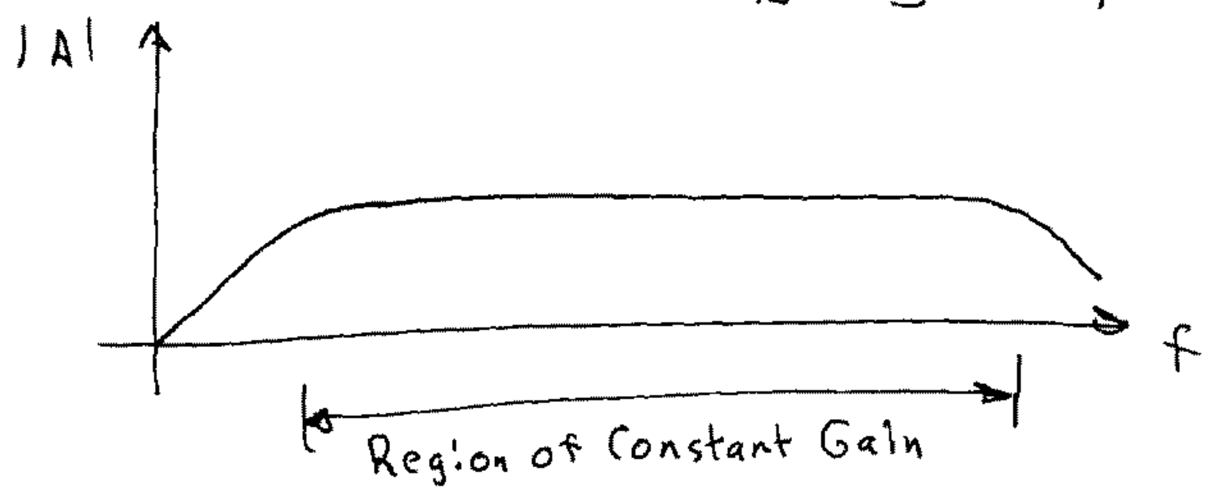
Input	Output	Type	Dimensions
V	V	Voltage	Dimensionless
I	I	Current	"
V	I	Transconductance	A/V
I	V	Transresistance	Ω

Amplifiers are generally not ideal
(but can be nearly ideal)

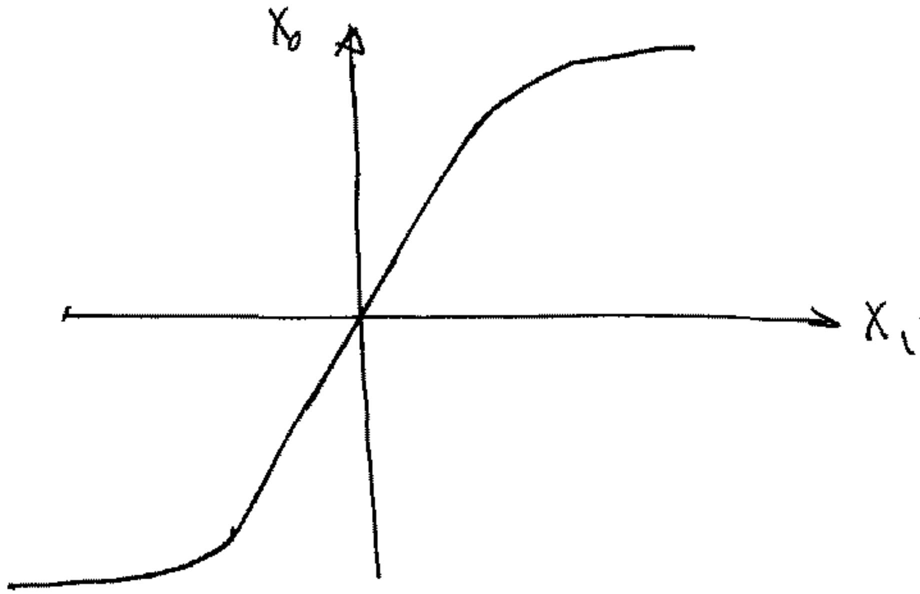
- Gain can be frequency dependent

$$\frac{X_o}{X_i} = K \longrightarrow \frac{X_o}{X_i} = A(s)$$

  - for good practical amplifiers, $A(s)$ will be nearly constant over a wide frequency range



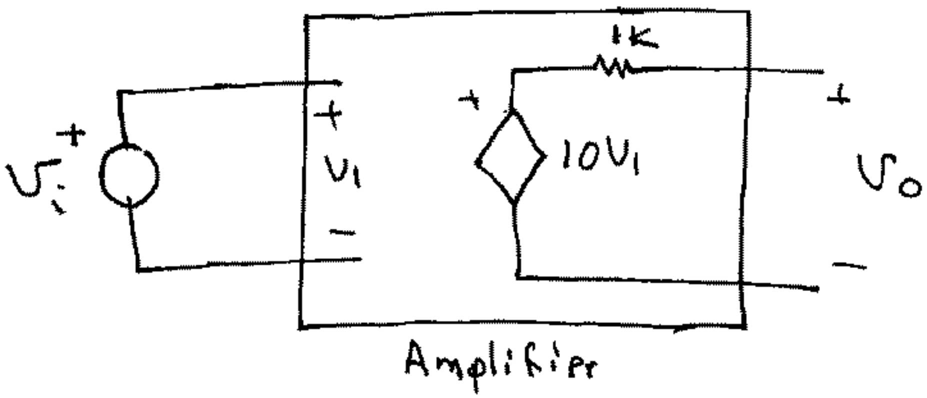
- The amplifier will display some nonlinearity



For a practical amplifier, the output range over which the amplifier is linear or nearly linear can be quite large

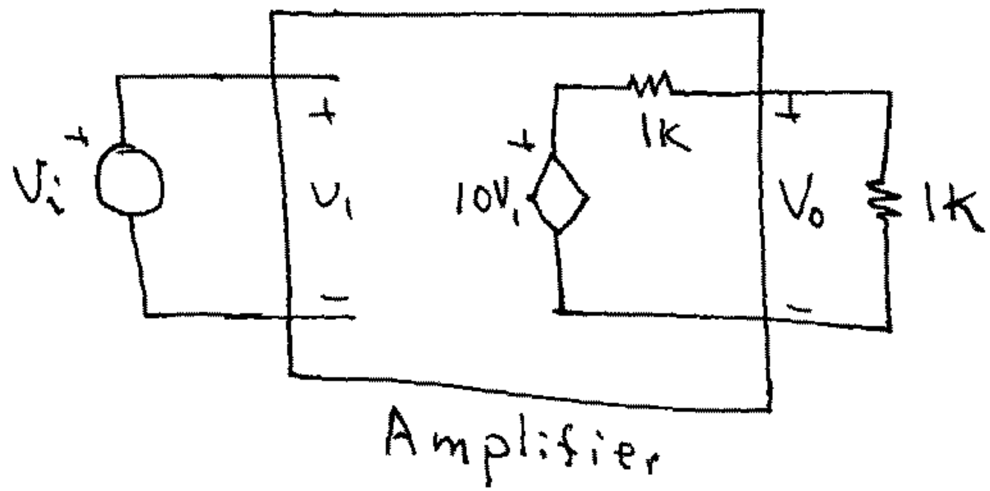
- The input and output impedances may not be ideal

Example: Voltage Amplifier, ideally $V_o = K V_i$



$v_o = 10 v_i$

Gain $= 10$

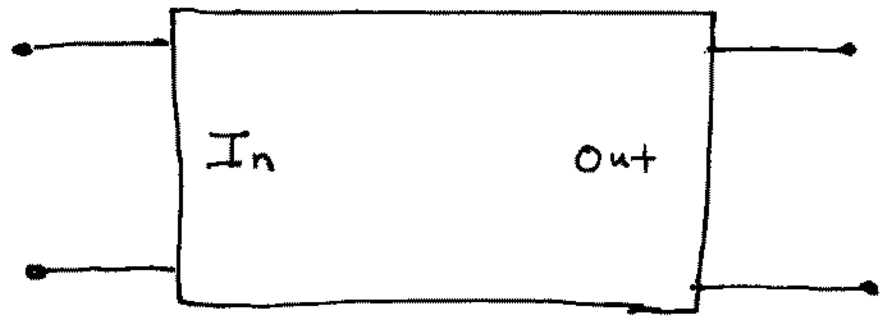


$$v_o = \left(\frac{1K}{1K+1K}\right) 10 v_i$$

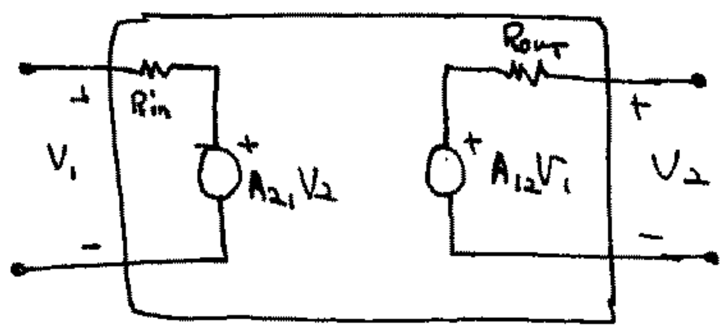
$$v_o = 5 v_i$$

Gain dependent upon Load ☹

Amplifiers are Two-Port Networks



General Two-Port



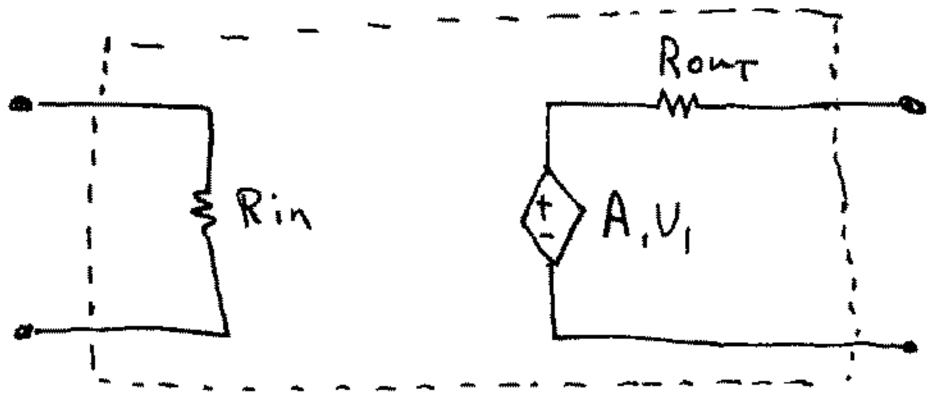
Linear Two-Port Model

Rin : input impedance

Rout : output impedance

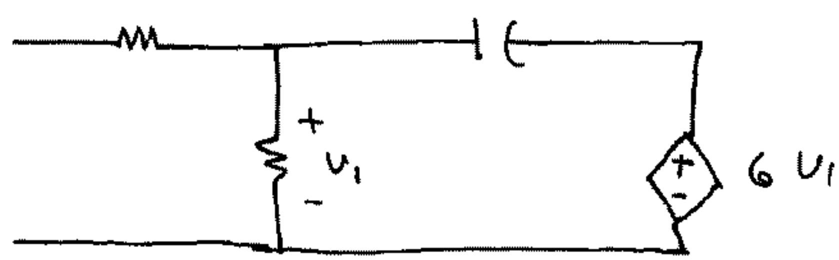
Amplifiers Ideally Unilateral

— Signals propagate in only one direction

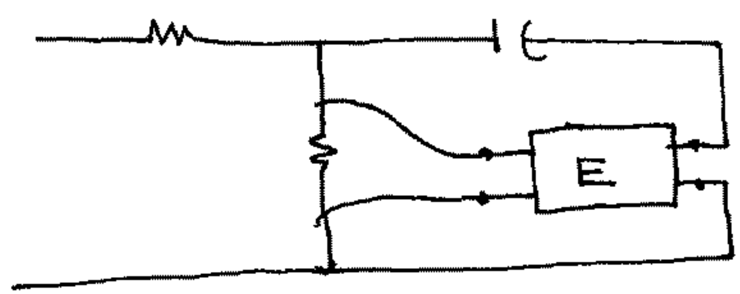


- Dependent sources discussed in EE 201 were actually two-ports but this terminology may not have been used.
- Dependent sources used in SPICE are two-ports
- Dependent sources are <u>amplifiers</u>.

Dependent Source Representation from EE 201



Two-port Representation of Dependent Source



Consider 4 basic Amplifiers

1) Voltage

2) Current

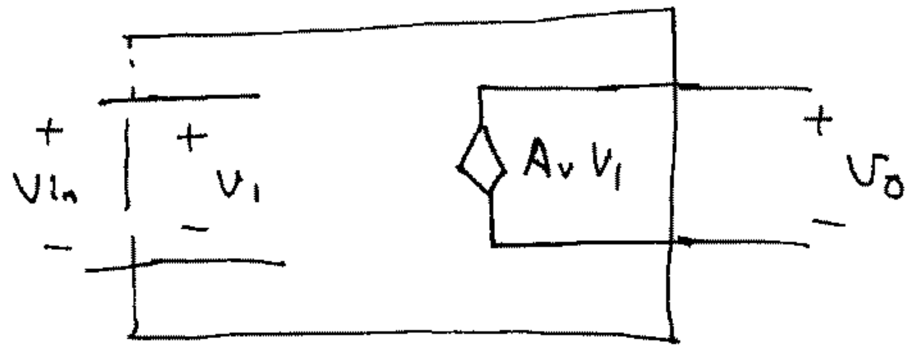
3) Transconductance

4) Transresistance

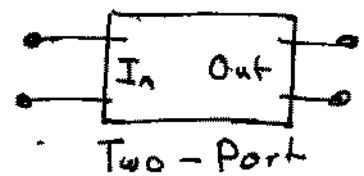
- Why so many different amplifier types?
  - because we can have them?
  - some transducers have output variables different than what is needed
  - sometimes perf. can be optimized by using a particular amplifier type
- If the amplifiers are not ideal, they are all functionally the same

Voltage Amplifier

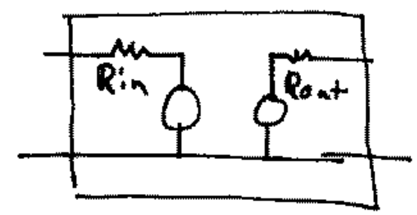
Ideal



- Note this is a two-port



- Both ports can be represented as a Thevenin equivalent circuit



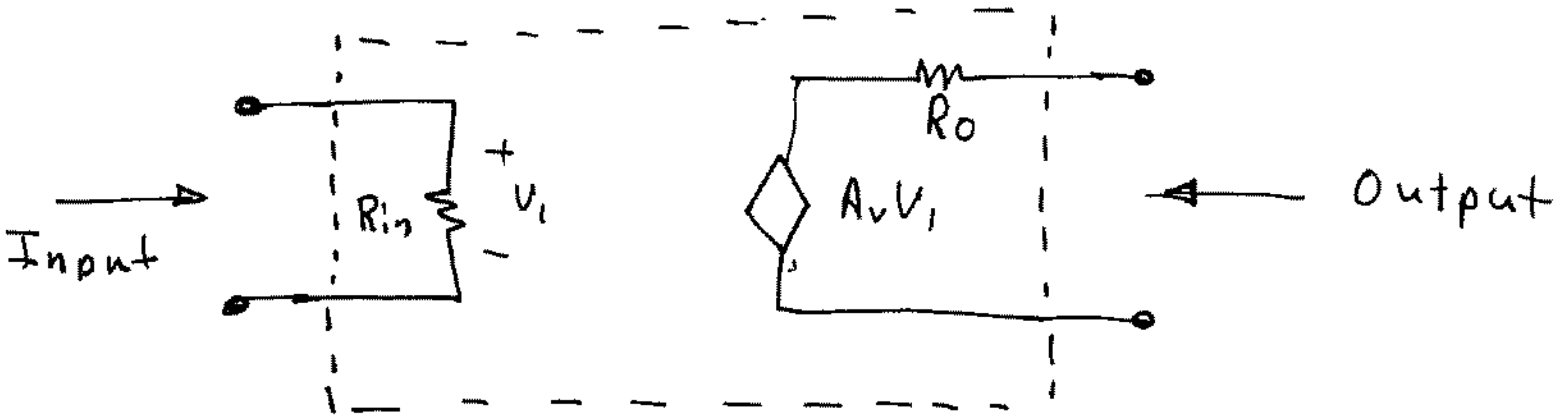
$R_{in} \overset{defn}{\equiv}$ Thevenin impedance at input port

$R_{out} \overset{defn}{\equiv}$ Thevenin impedance at output port

For ideal voltage amplifier

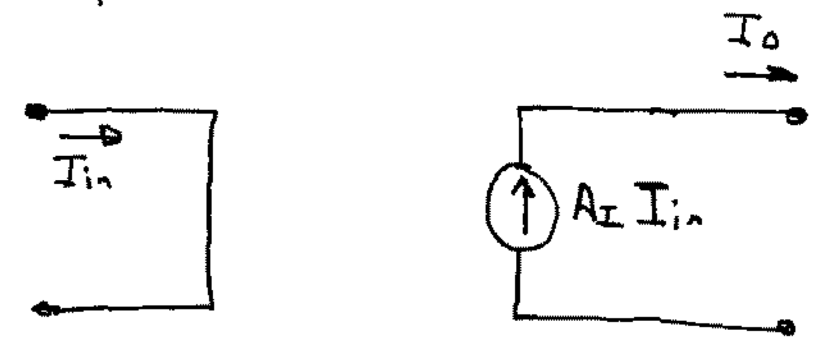
$$R_{in} = \infty \quad , \quad R_o = 0$$

Practical Voltage Amplifier



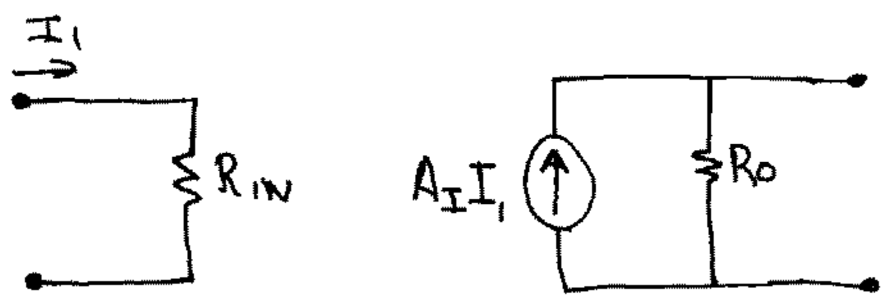
Current Amplifier

Ideal



$R_{in} = 0$ $\qquad R_{out} = \infty$ $\qquad \dfrac{I_o}{I_{in}} = A_I$

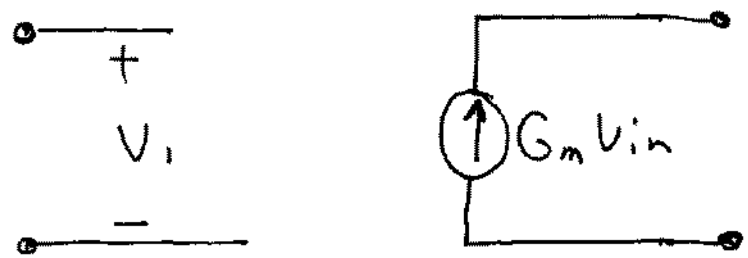
Practical



Note: Norton equivalent shown on output port

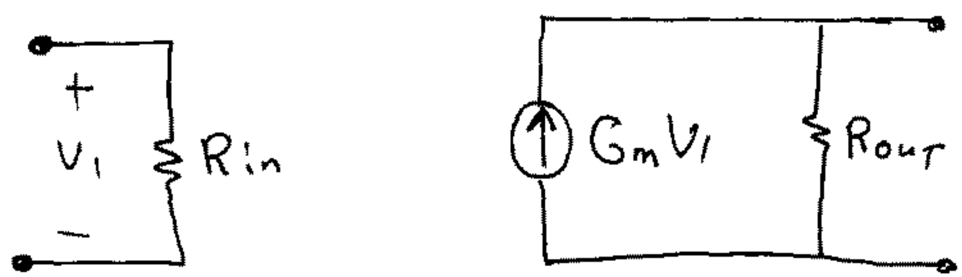
Transconductance Amplifier

Ideal



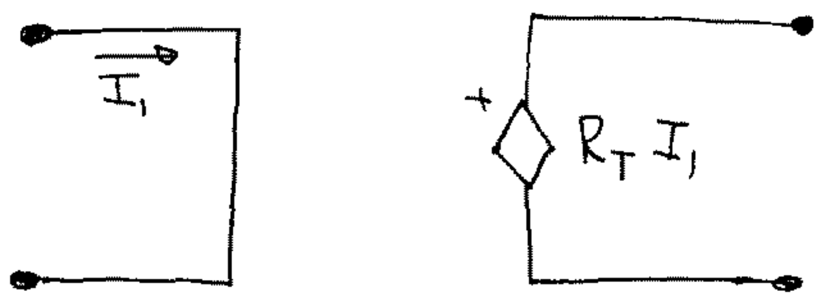
$$R_{IN} = \infty, \quad R_{OUT} = \infty$$

Practical



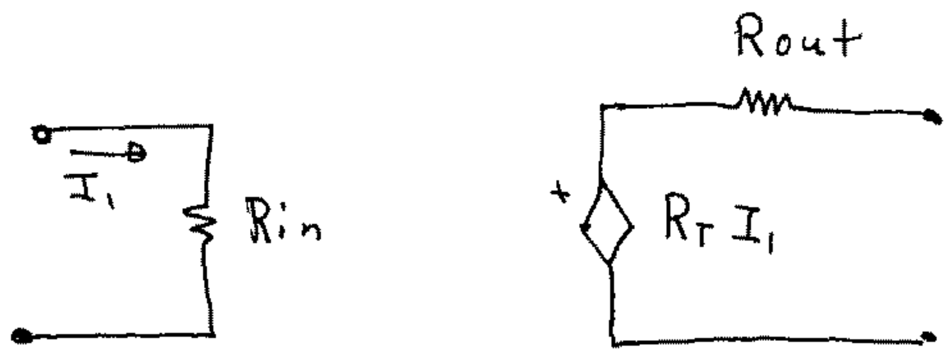
Transresistance Amplifier

Ideal



$R_{IN} = 0$ $R_{OUT} = 0$

Practical

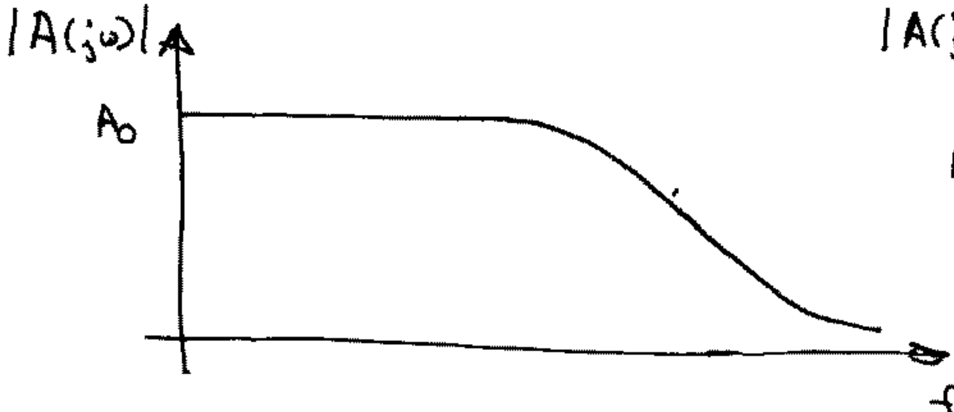


Ideal Port Impedances

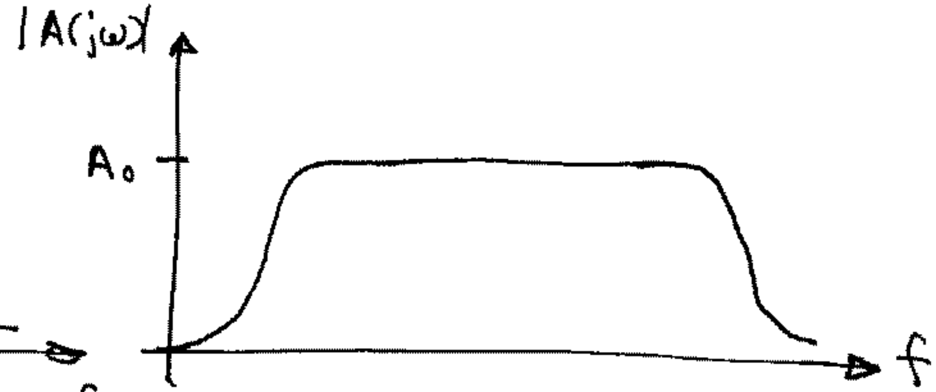
		R_{IN}	
		0	∞
R_{OUT}	0	R_T	A_V
	∞	A_I	G_M

Frequency Response of Amplifiers

- In a region of interest, amplifiers behave linearly and can be modeled by a transfer function $A(s)$
- All amplifiers exhibit a roll-off in gain at high frequencies and some also a rolloff at low frequencies

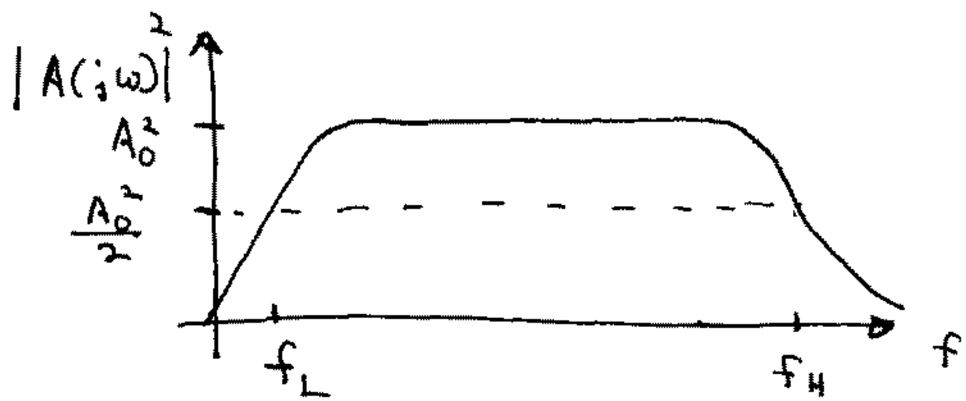


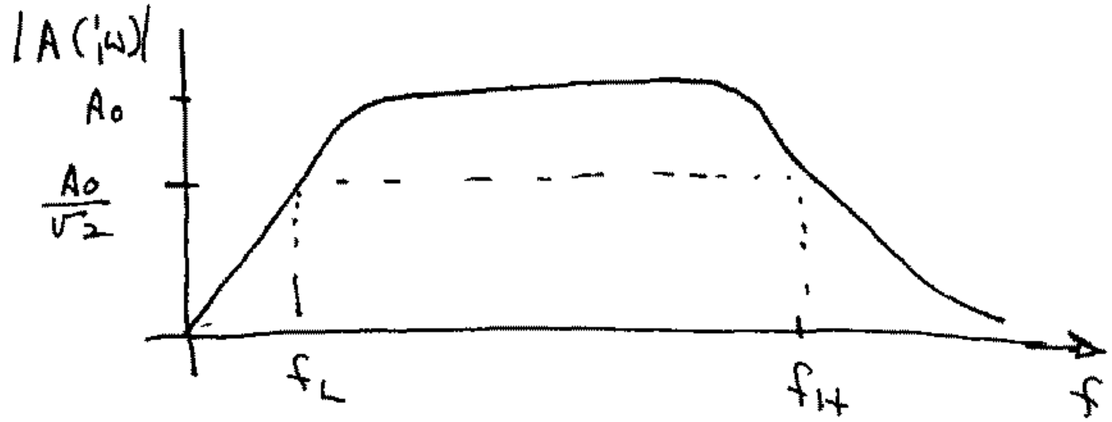
"Lowpass"



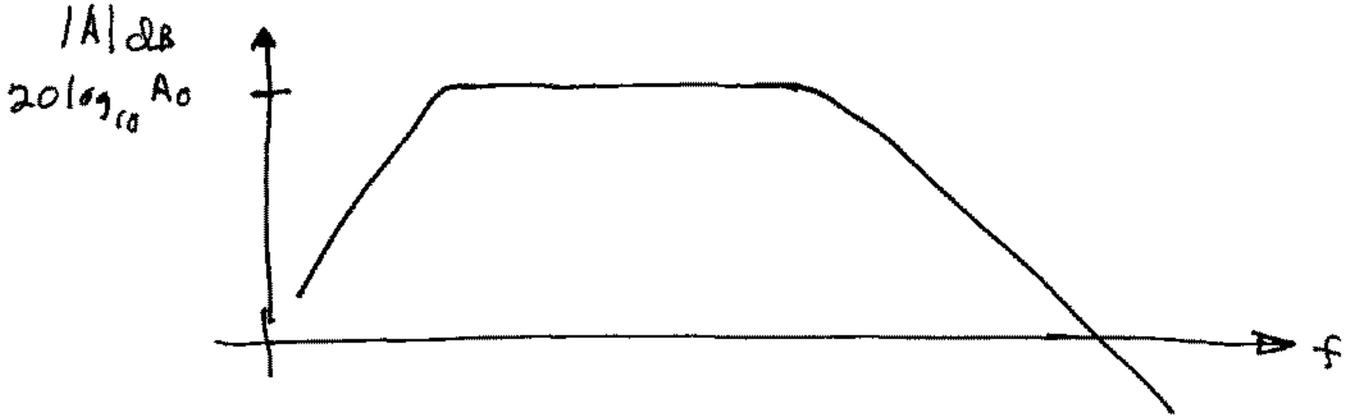
"Bandpass"

Half-power frequency is frequency where output power drops to ½ of the peak output power.





3-dB frequency notation



$$20\log_{10} A_o - 20\log_{10}\left(\frac{A_o}{\sqrt{2}}\right) = 20\log_{10} A_o - 20\log_{10} A_o + 20\log_{10}\sqrt{2}$$

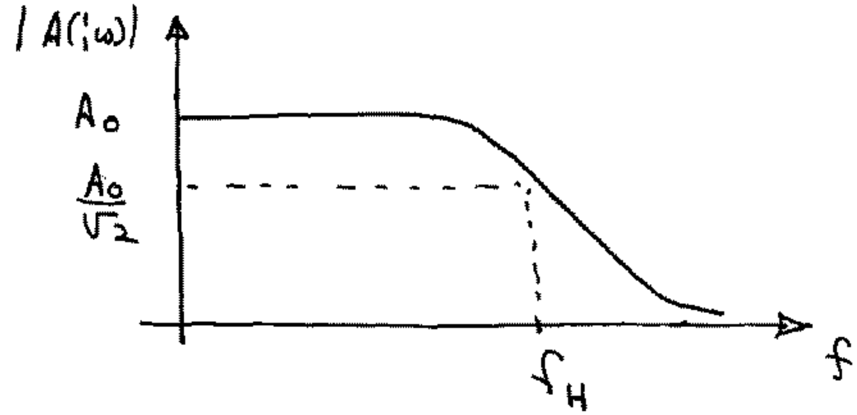
$$= 20\log_{10}\sqrt{2}$$

$$= 3.01\,\text{dB}$$

Half-power frequency often termed "3dB frequency" since it is close to a 3dB drop in magnitude

Typical $A(s)$

1) Lowpass



$\omega_H = 2\pi f_H$

$$A(s) \simeq \frac{A_o}{\frac{s}{p} + 1}$$

$$|A(j\omega)| = \frac{A_o}{\sqrt{1 + \frac{\omega^2}{p^2}}}$$

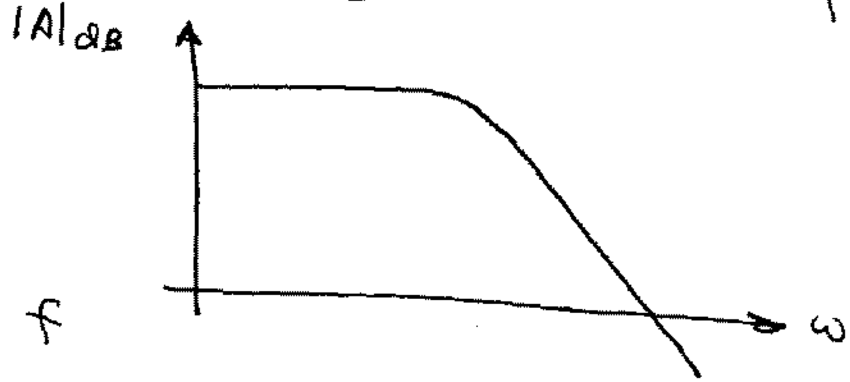
$$|A(j\omega_H)| = \frac{A_o}{\sqrt{2}} = \frac{A_o}{\sqrt{1 + \frac{\omega_H^2}{p^2}}}$$

solving, obtain $\omega_H = p$

$$|A(j\omega)|_{dB} = 20\log_{10}\left(\frac{A_o}{\sqrt{1+\frac{\omega^2}{p^2}}}\right)$$

at high f $\quad |A(j\omega)| \simeq \dfrac{A_o}{\omega/p} = \dfrac{A_o p}{\omega} = \dfrac{GB}{\omega}$

where GB is the product of gain and bandwidth
– termed gain-bandwidth product



At high f

$$20\log_{10}|A(j\omega)| \simeq 20\log_{10}\left(\frac{GB}{\omega}\right)$$

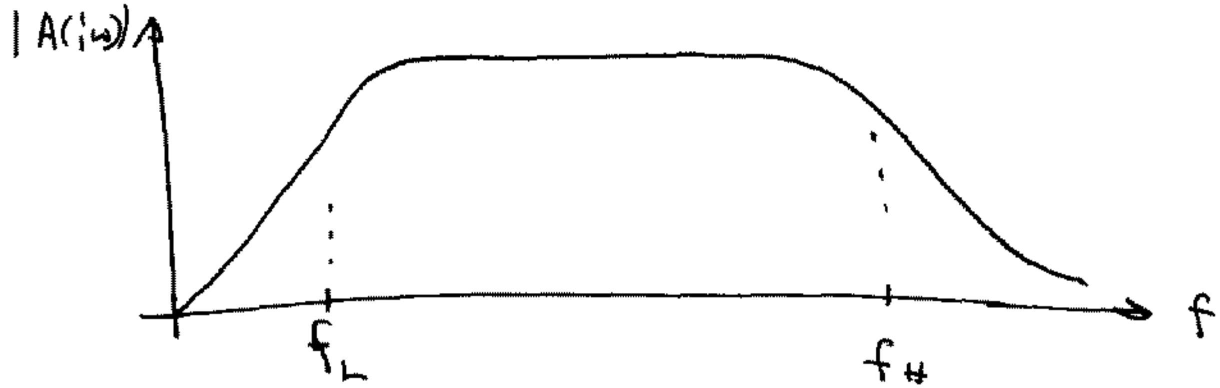
in 1 decade, $\Delta|A| = 20\log_{10}\frac{GB}{\omega} - 20\log_{10}\frac{GB}{10\omega} = -20dB$

$\therefore$ roll-off is 20dB/decade

or 6.02dB/octave

Typical A(s)

2) Bandpass



$\omega_L = 2\pi f_L$

$\omega_H = 2\pi f_H$

Typical A(s)

$$A(s) = \frac{A_0 \cdot s}{(s+P_1)\left(\frac{s}{P_2}+1\right)} \approx \begin{cases} \frac{A_0 s}{s+P_1} \simeq \frac{A_0 s}{P_1} & f < f_L \\ A_0 & f_L < f < f_H \\ \frac{A_0}{\left(\frac{s}{P_2}+1\right)} \simeq \frac{A_0 P_2}{s} & f > f_H \end{cases}$$

$\omega_L \simeq P_1$

$\omega_H \simeq P_2$

To find ω_L

$$A(s) = \frac{A_o s}{s + P_1}$$

$$|A(j\omega_L)| = \frac{A_o}{\sqrt{2}}$$

$$|A(j\omega)| \simeq \frac{A_o \omega}{\sqrt{\omega^2 + P_1^2}}$$

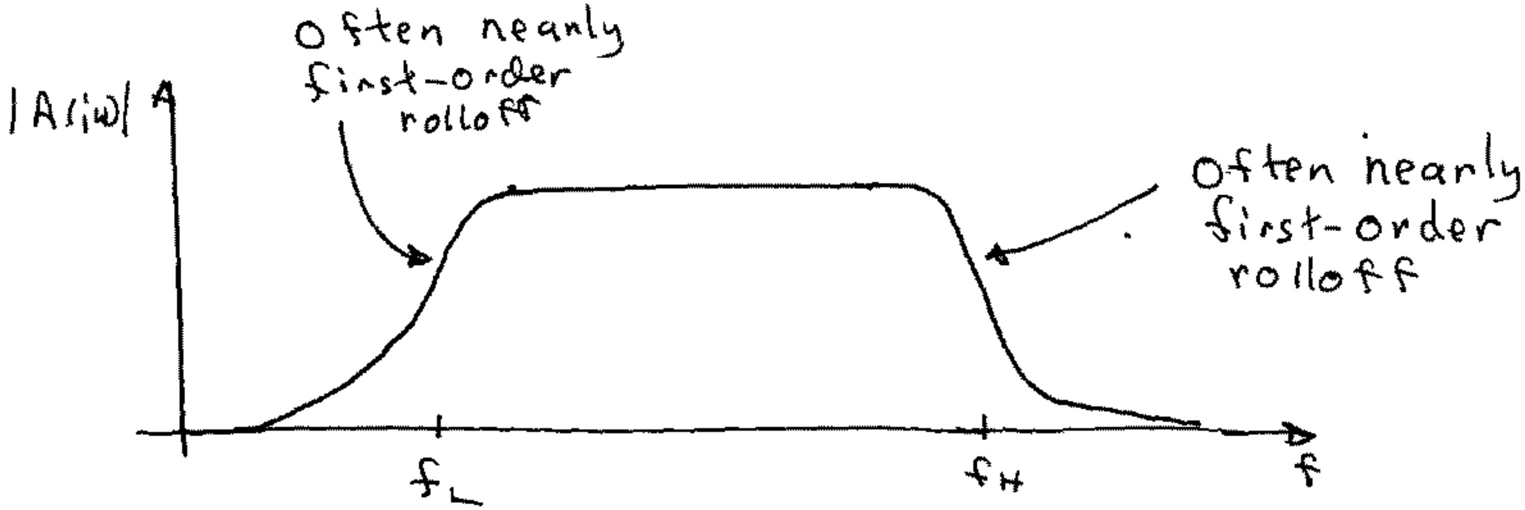
$$\therefore \quad \frac{A_o \omega_L}{\sqrt{\omega_L^2 + P_1^2}} = \frac{A_o}{\sqrt{2}}$$

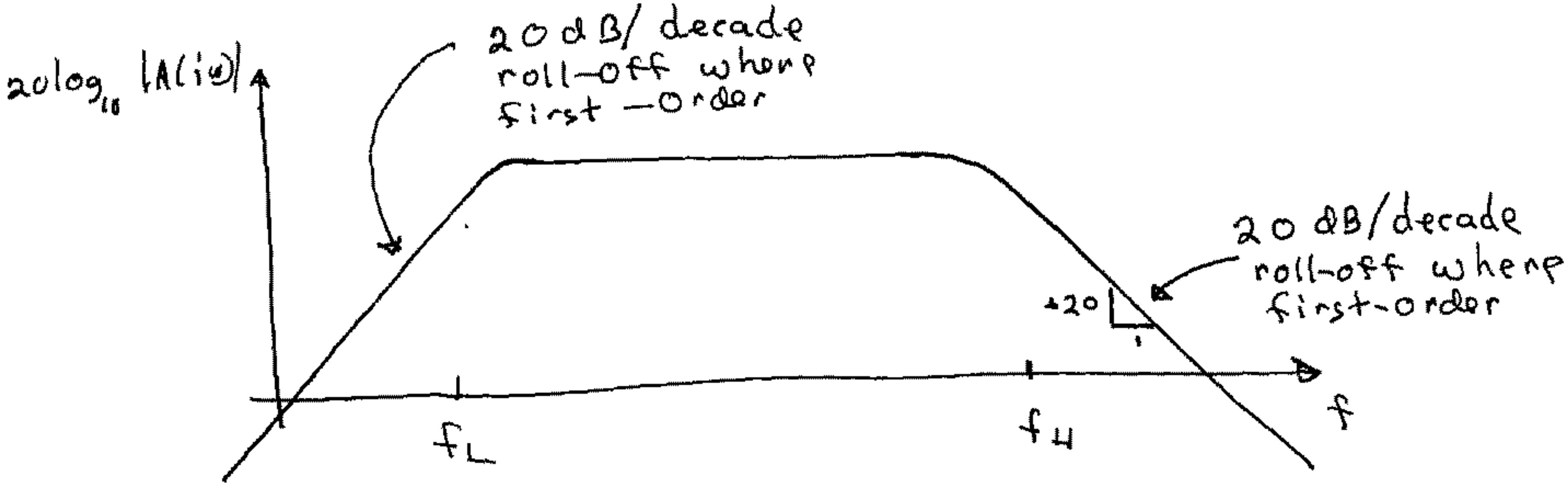
$$\frac{\omega_L^2}{\omega_L^2 + P_1^2} = \frac{1}{2}$$

$$2\omega_L^2 = \omega_L^2 + P_1^2$$

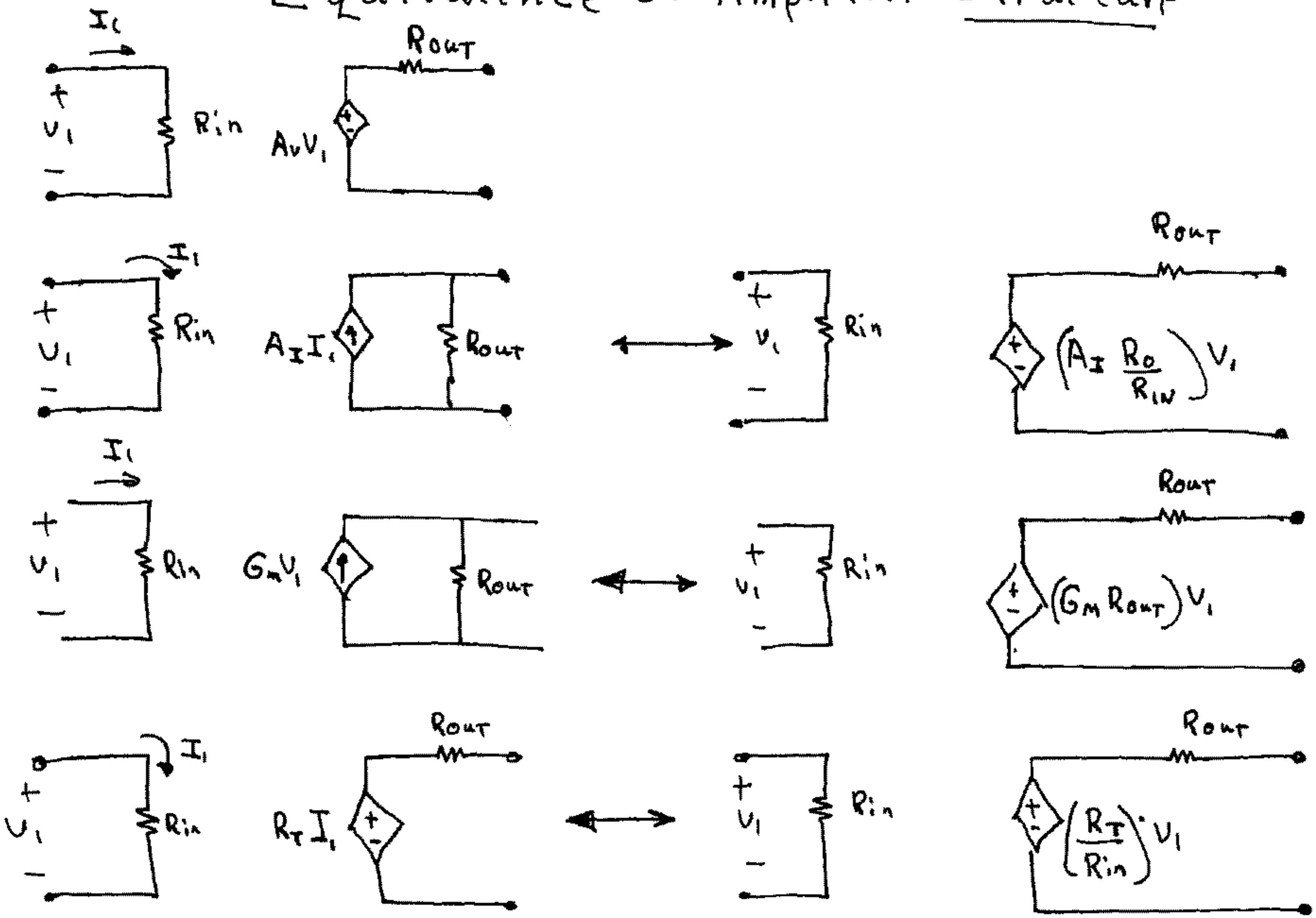
$$\omega_L = P_1$$

Linear vs Log Axis





Equivalence of Amplifier Structure



Relative magnitudes of R_{IN}, R_{OUT}, R_T & G_M determine which type is most ideal

End of Lecture ~~5~~